Tick Size, Lot Size, and Liquidity in Futures Trading

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Abstract

Futures are standardized and heavily regulated contracts, and these features make futures trading possible at liquid secondary markets. However, regulations constrain futures trading at discrete prices and quantities by imposing a minimum tick size and a minimum lot size. We show, theoretically and empirically, that the tick size and the lot size are important for futures trading costs. In our model, we express the futures bid—ask spread, given lot size and tick size restrictions, as a function of futures volatility and trading activity by informed and uninformed traders. Our empirical results support the theoretical model.

Key words: Futures, tick size, lot size, transactions costs

JEL classification: G12, G13, G14, G18, D47

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1. Introduction

Futures are standardized and heavily regulated contracts. These features enable futures trading in highly liquid secondary markets. However, some of the regulations might induce frictions in futures trading, causing unnecessarily low market liquidity and high trading costs. Futures markets' regulations constrain trading at discrete prices and quantities by imposing a minimum tick size and a minimum lot size. A too wide tick size, and a too large lot size, could impede futures traders from achieving optimal positions, whether they engage in arbitrage, hedging, or speculation. However, continuous pricing, or a too narrow tick size, could be suboptimal regarding market liquidity and total welfare. A too fine pricing grid could delay the negotiation and competition process, which increases trading costs (Anshuman and Kalay, 1998; Cordella and Foucault, 1999). A positive tick size produces rents for liquidity provision, which encourages market participants to undercut the best prices and is essential for a competitive market with time priority (Seppi, 1997; Cordella and Foucault, 1999; Graziani and Rindi, 2023).

To highlight the importance of the minimum tick size and lot size in the index futures market, consider one of the world's most actively traded equity index futures markets; the E-mini S&P 500 index futures. The E-mini futures have a minimum tick size of 0.25 index points and a minimum dollar lot size of 50 times the index. Thus, futures prices are always discrete at increments of $12.50, whereas the minimum dollar lot size varies with the index. For example, on December 27, 2021, when the S&P 500 index closed at $4,791, the minimum lot size equaled $119,775. A casual analysis of the tick size and lot size constraints in the E-mini futures contract shows that the former is often binding while the latter is not. In the data, the average E-mini futures bid—ask spread is very close to the minimum $12.50, which indicates that the minimum tick size puts a binding constraint on the bid—ask spread. The average E-mini futures trade size is, however, safely above the minimum lot size, making a quantity constraint less likely.

We argue that the tick size and the lot size are of first order importance for futures market liquidity, and, thus, for determining the costs of trading in the futures market. In the paper, we substantiate our argument by both theoretical and empirical analyses. In the first part, we develop a theoretical model of futures trading. The model extends the models by Budish, Cramton and Shim (2015) and Li and Ye (2023) to fit the futures market. Although the model is developed for stock index futures, it can easily be amended for other underlying assets. The model consists of decisions from the futures market regulator and the futures exchange, and highlights how market makers, informed traders, and uninformed traders interact. First, the regulator exogenously sets the tick size and lot size that prevail at the futures market. Then, market makers set a competitive bid—ask spread that considers the likelihoods of the arrivals of informed and uninformed traders. Finally, the futures exchange chooses the optimal futures price that minimizes the transactions costs faced by futures traders.

Although our model is very similar to the one in Li and Ye (2023), it differs from theirs in the following aspects. First, the futures price in our model relates to the underlying index value by the cost of carry and the index multiplier, while the stock price in the model by Li and Ye (2023) relates to the value of the firm and the number of stocks. Second, we assume that each uninformed trader wants to trade futures that amount to x index units, while Li and Ye (2023) assume that each uninformed trader wants to trade a fraction of the firm's outstanding shares h. In futures markets, new futures contracts are created and added to open interest when trading occurs. In equity markets, the number of outstanding shares of a firm is fixed. Our uninformed traders arrive at a rate that depends on x and the order size it chooses. The corresponding arrival rate in Li and Ye (2023) depends on h. Third, Li and Ye (2023) assume that each uninformed trader is fast enough to trade its entire h without informed traders being able to "cut in between". We instead assume that an uninformed trader can only trade one order size of futures contracts, and not the entire x, at a time. In this sense, the market makers in our model are more exposed to informed trading than in the model by Li and Ye (2023).[2]

The key trade-off in the model is between discrete price and discrete quantity, so that at a high futures price, the futures price is relatively more continuous while the futures quantity is relatively more discrete than at a low future price. That is, when the futures price is high rather than low, the lot size restriction is causing relatively more friction than the tick size restriction.

The model produces several results. First, the model generates a decomposition of the futures bid—ask spread into one component that is driven by the tick size restriction and another component that is driven by the lot size restriction. The tick size driven component is equal to the minimum tick size, which makes it easy to perform an empirical decomposition of the bid-ask spread.

Second, given the tick size and lot size restrictions, the futures exchange chooses the optimal futures price that minimizes futures transactions costs by balancing the two restrictions. From a policy perspective, in a futures market where both restrictions are present, it is important for a futures exchange to take the trade-off into account when considering a change in the futures price (e.g., performing an index "split", or changing the index multiplier in index futures

[2] Note that we do not explicitly make assumptions about the speed of arrival of each type of trader. However, the model includes the arrival intensity of each type of trader that determines the likelihood of arrival.

markets). Accordingly, a price reduction would decrease transactions costs through the lot size restriction channel but increase transactions costs through the tick size channel.

Third, as an output from the model, we obtain an expression for the futures bid—ask spread, given lot size and tick size restrictions, as a function of futures volatility and trading activity by both informed and uninformed traders. The variables are straightforward to obtain in futures data, which enable us to evaluate the empirical accuracy of the model.

In the second part of the paper, we empirically test the model using data from the OMXS 30 index futures that are traded on Nasdaq Stockholm. We use regression analysis to estimate the relationship between the futures bid—ask spread, futures volatility, and trading activity, implied by the model. The empirical results strongly support the model.

The empirical results show that the average absolute futures bid—ask spread equals 0.27 SEK, which is very close to the minimum tick size that equals 0.25 SEK. The average trade size equals about six futures contracts, which is six times as large as the one-contract minimum lot size. Hence, we conclude that the tick size restriction is more severely impacting futures trading than the lot size restriction. Given the levels of tick size and lot size, our policy suggestion would be for the futures exchange to increase the futures price (index multiplier) until the effects from the two restrictions balance. Note that our model is flexible in the sense that it can be used for policy decisions irrespective of the actual market conditions, i.e., regardless of if the tick size restriction is more severe than the lot size restriction, or vice versa.

Several studies construct theoretical models to examine the optimal tick size and its effects on market liquidity. Most previous papers study the discrete price restriction, the tick size, in isolation (e.g., Seppi, 1997; Anshuman and Kalay, 1998; Cordella and Foucault, 1999; Graziani and Rindi, 2023), while Li and Ye (2023) analyze the restrictions from discrete price and quantity, and how the trade-off between them affect transactions costs and the optimal tick size. Li and Ye (2023) focus on a model for the equity market, and we extend their model to fit the futures market.

The literature contains many empirical studies of tick size changes in equity markets (Angel, 1997; Badicore, 1997; Bessembinder, 2003; Goldstein and Kavajecz, 2000, Albuquerque, Song, and Yao, 2020; Werner, Rindi, Buti and Wen, 2022; Li and Ye, 2023) and in futures markets (Bollen, Smith and Whaley, 2003; Kurov and Zabotina, 2005; Kurov, 2008; Alampieski and Lepone, 2009; Martineza and Tse, 2019). For example, Bollen et al. (2003) analyze the event from 1997, when the Chicago Mercantile Exchange reduced the lot size, and simultaneously doubled the minimum tick size, of the S&P 500 futures contract. They find that the changes reduced liquidity and lowered trading activity. Although Bollen et al. (2003) do not study the tradeoff between tick size and lot size, their results show that the futures bid—ask spread was binding before the regulatory changes, while the lot size was not. According to our theoretical model, the optimal changes would be the opposite of those implemented. Thus, based on our model, the policy recommendations to regulators would be to lower the minimum tick size, and/or increase the lot size, if, indeed, the aim of the regulatory changes would be to increase futures market liquidity.

Modern financial markets are to various extent dominated by high-frequency traders (HFTs). In the model by Budish et al. (2015), HFTs are acting as both market makers and informed traders, so called "snipers". Li and Ye (2023) do not explicitly include HFTs in their model. Instead, they state that under the assumption that the information becomes common knowledge as soon as it is traded on by informed traders, their general population of market makers and informed traders is equivalent to the setup in Budish et al. (2015).[3] Hence, theoretically, our model setup is consistent with both a modern market where HFTs dominate liquidity supply, and a more traditional market in which designated market makers supply liquidity.

Empirically, Hagströmer and Nordén (2013) find that HFTs are responsible for almost 30% of equity market trading volume. At the equity index futures market, Hou, Nordén and Xu (2024) find that HFTs account for 36% of the traded futures contracts. Hagströmer and Nordén (2013) also report that HFTs are engaged in both liquidity supply and demand, with the lion share of their trading volume (63-72%) resulting from market making activity. Given the importance of HFTs in modern financial markets, we investigate if HFT activity affects our empirical results. We find that the empirical results support the model when HFT activity, relative to total futures activity, is both high and low.

2. Theory and empirical framework

In this section, we present a model of futures trading based on the models of Budish et al. (2015) and Li and Ye (2023). We adapt their models for the futures market to illustrate how the minimum tick size restriction and the minimum lot size restriction affect futures trading and transactions costs. In the following, we first go through the model and derive an expression for the futures bid—ask spread. Second, we derive a testable regression equation for the futures bid—ask spread as a function of observable variables.

2.1 A model of futures trading with tick size and lot size restrictions

[3] Under this assumption, informed traders can adversely select market makers only once per piece of information.

The model consists of three stages in which the regulator, the futures exchange, and traders sequentially make decisions. At stage 1, the regulator sets the minimum tick size Δ and the minimum lot size L. We do not model the regulator decision. Instead, we treat Δ and L as exogenously given at discrete levels as in the main analysis of Li and Ye (2023). In index futures markets, the minimum tick size and the minimum lot size are expressed in terms of the underlying index. For example, for the E-mini S&P 500 futures, the tick size is 0.25 index points and the lot size is 50 times the index.

The underlying index value is v at stage 2. Given the tick size and lot size regulations, the futures exchange chooses the contract multiplier k so that the futures contract value relates to the index value as $p = vc/k$, where c equals the cost of carrying the underlying index stocks until futures maturity. The multiplier k makes it possible for the exchange to split the index, while c satisfies the no-arbitrage conditions between the futures price and the index value.[4]

The futures market opens at stage 3. Let v_s denote the index value at time s. The index value, and, thus, the futures price, depends on public news and private information about the underlying index stocks, and we assume that v_s evolves continuously as a Poisson process with jump intensity λ_J, and jump sizes σv_s and $-\sigma v_s$ occurring with equal probability. Since the index value jumps up or down with σ%, the futures price jumps up or down at the same rate σ% and with the same intensity λ_J.[5] We assume that the exchange's objective is to maximize futures liquidity,

[4] Note the different setup compared to the model by Li and Ye (2023). We focus on futures pricing, where the futures price relates to the index value v by the factor c/k, where c is the cost of carry and k is the contract multiplier. Li and Ye (2023) focus on the stock price, which relates to firm value divided by the number of outstanding shares h.
[5] After a jump, the index value is either $v_s(1+\sigma)$ or $v_s(1-\sigma)$. Since $p_s = v_s c/k$, the corresponding futures price is either $p_s(1+\sigma)$ or $p_s(1-\sigma)$ after a jump.

which is the same as to minimize futures transactions costs. This is accomplished by optimally choosing k, and thus p.

Three different types of traders arrive at stage 3: a market maker, uninformed traders, and informed traders.[6] The market maker sets competitive bid—ask quotes at which other traders can trade. The informed traders know each futures price before it jumps up or down and are therefore able to adversely select the market maker's quotes. We assume that informed traders can only adversely select one futures lot before the market maker reacts and updates her quotes. The informed traders will trade only when it is profitable to do so, i.e., when the jump in the futures price is larger than half of the bid—ask spread set by the market maker.

Uninformed traders arrive randomly at a Poisson process with jump intensity λ_I. We assume that each uninformed trader wants to trade a parent order of futures that amounts to x index units, so that xv is the desired dollar trading volume. The parent order could be split into child orders with order size qL, where pqL is the dollar order size. Uninformed traders choose q to minimize the transaction costs of the parent orders. We assume that the arrival intensity $\lambda_I = xv/pqL$.[7] Thus, given that x, v, p, and L are exogenous to the uninformed traders, the choice of q determines their order arrival rate. For example, a low q requires the trader to arrive more frequently to trade the parent order x. The market maker observes q and sets bid—ask quotes B_t^q and A_t^q for futures lots of size qL that are multiples of Δ.

[6] Budish et al. (2015) allow several HFTs to act as both market makers and "snipers", while Li and Ye (2023) populate their model with one market maker and "informed traders". We choose a similar setup as Li and Ye (2023). For the theoretical results of this paper, it does not matter if our model has one or several market makers. Thus, the model is appropriate for describing the mechanisms of trading in both a "traditional" designated market maker setting and in a "modern" market with voluntary liquidity supply, often dominated by HFTs.

[7] The arrival intensity of each uninformed trader is assumed to depend on the number of index units it wants to trade (x), and the order size (qL) it chooses. Li and Ye (2023) assume that the dollar arrival rate of uninformed traders is a fraction (h) of the firm value.

Li and Ye (2023) show that the market maker provides liquidity of qL futures contracts at prices $p_t \pm S_t^{tot}/2$, where S_t^{tot} is the nominal bid—ask spread. They also show that, in expectation, it can be decomposed into a lot-driven component S_t^L and a tick-driven component S_t^Δ as

$$E(S_t^{tot}) = E(S_t^L + S_t^\Delta) = \frac{2\sigma\lambda_S}{\lambda_I + \lambda_S} p_t + \Delta, \tag{1}$$

where $\lambda_S = \lambda_J Pr(\sigma p_t > S_t^{tot}/2)$ is the arrival intensity of informed traders, and $Pr(\sigma p_t > S_t^{tot}/2)$ denotes the likelihood that the jump size is large enough to make the information profitable.

Note that the lot-driven component, which is the first term on the right-hand side of Eq. (1), is equivalent to the bid—ask spread obtained in the model by Budish et al. (2015), that assumes a continuous price (no tick size restriction) and a lot size L = 1. There are no informed traders per se in the model by Budish et al. (2015). Instead, snipers are fast enough to react to public information to adversely select market makers' quotes. For our purposes, λ_S includes the arrival of either snipers that react to public information, or privately informed traders. Note also that the tick size driven component is equal to Δ in expectation.[8]

The bid—ask spread is increasing in both the jump size σ and the arrival intensity of informed traders λ_S but decreasing in the arrival intensity of uninformed traders λ_I. Remembering that $\lambda_I = xv/p_t qL$, the bid—ask spread is smaller for more frequent uninformed order arrival, i.e., for a smaller value of $p_t qL$. Hence, the spread is affected by the regulator's choice of Δ and L, the uninformed traders' choice of q, and the exchange's choice of p (through the choice of k).

[8] See Appendix A for a derivation of the two components of the bid-ask spread in Eq. (1).

In equilibrium, the exchange chooses the optimal price that minimizes transaction costs. Following Li and Ye (2023), we define the expected futures execution costs as:

$$E\left[\frac{S_t^{tot}}{2p_t}(\lambda_I p_t qL + \lambda_S p_t qL)\right] = \sigma\lambda_S p_t qL + \frac{\Delta xv}{2p_t} + \frac{\Delta\lambda_S qL}{2}, \tag{2}$$

where $S_t^{tot}/2p_t$ is half the percentage bid—ask spread, relative to the futures price, $(\lambda_I p_t qL + \lambda_S p_t qL)$ is the traded volume from either informed or uninformed traders as, and the equality follows from solving for the expected bid—ask spread in Eq. (1).

From Eq. (2), we note that the expected execution cost is increasing in both the minimum tick size Δ and the minimum lot size L. Moreover, the first term on the right-hand side of the equality sign in Eq. (2) $(\sigma\lambda_S p_t qL)$ is driven by the lot size, the second term $(\Delta xv/2p_t)$ is driven by the tick size, and the third term $(\Delta\lambda_S qL/2)$ by both. Notice the trade-off between the tick-size- and the lot-size-driven effects from a change in price on the expected execution costs. An increase in the price would increase the part driven by the lot size and decrease the part driven by the tick size.

We solve for the p_t^* that minimizes the expected execution costs according to:

$$\min_p \left[\sigma\lambda_S p_t qL + \frac{\Delta xv}{2p_t} + \frac{\Delta\lambda_S qL}{2}\right]. \tag{3}$$

In the solution, the exchange minimizes transaction costs by balancing the lot size restriction with the tick size restriction, and chooses the price that solves $\sigma\lambda_S p_t qL = \Delta xv/2p_t$, that is,

$$p_t^* = \sqrt{\frac{\Delta xv}{2\sigma qL\lambda_S}} = \frac{\Delta\lambda_I}{2\sigma\lambda_S}, \tag{4}$$

where the second equality in Eq. (3) holds given the definition of $\lambda_I = xv/p_t qL$.

The resulting p_t^* in Eq. (3) is the optimal futures price that the exchange can choose, given the tradeoff between discrete price and discrete quantity. By substituting Eq. (3) into Eq. (1), we get a corresponding equilibrium expression for the optimal bid—ask spread as:

$$E^*(S_t^{tot}) = \Delta\left(1 + \frac{\lambda_I}{\lambda_I + \lambda_S}\right). \tag{5}$$

In the model, the market maker sets the bid—ask spread to compensate for the adverse selection risk of trading with informed investors. If the minimum tick size is large enough to cover the adverse selection risk, it is binding the competitive bid—ask spread at one tick. Otherwise, if the minimum tick size is too small to compensate for the adverse selection risk, the competitive bid—ask spread is wider than the minimum tick size, which is not binding. Hence, whether the minimum tick size is binding or not depends on how large it is relative to the adverse selection risk.

The market maker also takes the order size, and, thus, the arrival rate, of the uninformed traders into account when quoting the bid—ask spread. The minimum lot size is binding for the uninformed traders' order size if they wish to trade in smaller sizes than one lot.

The expected execution costs in Eq. (2) increase in both the minimum tick size Δ and the minimum lot size L. Note that our purpose is not to identify optimal levels for Δ and L. Instead, we assume that Δ and L are exogenously determined, and that the futures exchange can mitigate their effects on execution costs by choosing the optimal futures price. This is true even if one, or both, of the minimum tick size and the minimum lot size is binding. For example, if the exchange chooses a larger futures price, the impact from the tick size on execution costs becomes relatively smaller, and the minimum tick size is less likely to be binding the bid—ask spread.

However, with a larger futures price, the lot size has a relatively larger impact on execution costs and is more likely to be binding uninformed investors' trade size.

2.2 Policy implications

The model provides a roadmap for exchange officials and regulators to follow when they seek to increase futures market liquidity. The futures price is the key variable for the exchange. In both the model and in practice, the exchange should adjust the index multiplier k to minimize transactions costs. If all variables in Eq. (3) are known, it is easy to choose the correct optimal futures price. In reality, most of the variables are not known and/or difficult to estimate. Thus, we recommend that a futures exchange instead uses our model as an indicator of whether it is optimal to increase or decrease the futures price by adjusting the index multiplier. If, e.g., the tick size is binding while the lot size is not, our model signals that it is optimal to increase the futures price. The price increase will lead to an increase in the lot size driven part of transactions costs in Eq. (2), and a decrease in the corresponding tick size driven part. Importantly, however, the overall effect will be a decrease in transactions costs since the futures price is closer to an optimal level.

The tick size and the lot size are exogenous in our model. Nevertheless, the model is important to a regulator who is responsible for setting tick size and lot size. In a futures market where the tick size is binding and the lot size is not, the regulator could lower the tick size to set the stage for lower transactions costs with a better balance between the tick-driven and lot-driven cost parts. In the case that the lot size is binding, and the tick size is not, the regulator could correspondingly lower the lot size to minimize transactions costs.

2.3 Regression analysis

By rearranging Eq. (1) and taking the natural logarithm of both sides we get:

$$log\left(\frac{E(S_t^{tot}) - \Delta}{p_t}\right) - log(\sigma\lambda_S) = -log(\lambda_I + \lambda_S) + log(2), \tag{6}$$

where the tick size adjusted relative expected bid—ask spread, minus the term related to the jump parameters $\sigma\lambda_S$, is negatively related to the arrival intensity of both uninformed and informed traders $\lambda_I + \lambda_S$.

To test the relationship in Eq. (5) empirically, we estimate the expected futures bid—ask spread in an interval τ as the observed average during the interval. Moreover, we hypothesize that the average logarithm of the tick size adjusted relative bid—ask spread in the interval τ, $log(RSP_\tau)$, after deducting the variance of index returns $log(Variance_\tau)$, is negatively related to the total number of futures trades $log(Trades_\tau)$, in the same interval τ.

We use the average five-second quoted futures bid—ask spread in the interval τ to construct the variable $log(RSP_\tau)$ by deducting the minimum tick size, dividing by the concurrent midpoint of the quoted spread, and taking the natural logarithm. Since σ and λ_S together determine the variance of the underlying index in the model, we argue that the variance of index returns approximates the term $log(\sigma\lambda_S)$. We estimate $log(Variance_\tau)$ with the natural logarithm of the realized variance of five-second index returns during the interval τ. In addition, we estimate the variable $log(Trades_\tau)$ with the natural logarithm of the total number of trades during the interval τ. Accordingly, we can rewrite the model in Eq. (6) as:

$$log(RSP_\tau) - log(Variance_\tau) = -log(Trades_\tau) + log(2). \tag{7}$$

In principle, the model in Eq. (7) could be estimated with regression analysis of $log(RSP_\tau)$ on $log(Variance_\tau)$ and $log(Trades_\tau)$, with a suitable interval length τ, say, e.g., five minutes. However, we note at least three obstacles with this empirical approach. First, the two variables $log(Variance_\tau)$ and $log(Trades_\tau)$ are highly correlated, which would lead to severe multicollinearity issues. Second, the variable $log(RSP_\tau)$ is persistent, and unlikely to be stationary. Third, the standard errors of the regression estimators would be large if the regression is run across observations at τ intervals. We deal with these obstacles by first: regressing the variance-adjusted relative bid-ask spread, $log(RSP_\tau) - log(Variance_\tau)$ on the variable $log(Trades_\tau)$, second: transforming the variables into first-differences, and third: aggregating the observations into daily and intraday averages according to the remedy in Andersen et al. (2018).

The regression equation is:

$$d(log(RSP_\tau) - log(Variance_\tau)) = \beta_1 d(log(Trades_\tau)) + \beta_0 + \varepsilon_\tau, \qquad (8)$$

where $d(.)$ denotes the difference between the variable in question in interval τ and interval $\tau - 1$, and ε_τ is a residual. A strict interpretation of the model, given that the empirical variable proxies perfectly capture their theoretical equivalents, impose the restrictions on the coefficients that $\beta_1 = -1$ and $\beta_0 = 0$. Thus, we interpret the empirical exercise as a goodness of fit test for our model and formulate statistical tests of hypotheses that the restrictions hold. If, on the one hand, we cannot reject the hypotheses that the model is good in the sense that it describes well how the frictions caused by the tick size and the lot size affect the bid—ask spread and transactions costs. Then, we have empirical support for the theoretical model, which further validates the policy recommendations that it produces.[9]

If, on the other hand, we can reject the hypotheses that the restrictions hold, the model would not work in the sense that either the actual bid—ask spread would over- or underestimate the effects from the frictions, or the model is simply wrong. In this case, we would need to be more careful when making policy recommendations based on the model.

3. Institutional setting and data

3.1 Description of the OMXS futures market

The Nasdaq Stockholm facilitates trading of the OMXS 30 index futures, where the index comprises the 30 largest and most actively traded stocks on the exchange, updated every six months. Futures trading for the OMXS 30 index is primarily concentrated on Nasdaq. Although it is technically feasible to trade these futures on other platforms, such occurrences are infrequent within the sample period being examined. Our analyses include the futures transactions on Nasdaq, and we exclude all other transactions (0.44%).

The market uses an electronic limit order book that allows traders to submit market or limit orders, which are executed based on the price-visibility-time priority rule. Only exchange members, including dealers and market makers, can directly trade on Nasdaq Stockholm. As is the case for most limit order book markets, any exchange member is allowed to provide liquidity by posting bid and ask quotes.

[9] For robustness, we also regress $d(log(RSP_\tau))$ on $d(log(Variance_\tau) - log(Trades_\tau))$. This alternative regression specification produces results that are the same empirical results as the ones from the regression according to Eq. (8).

Trading hours for the futures market start with an opening call auction at 8:45 AM and run until the closing call auction at 5:25 PM, or 12:55 PM on the day before a Swedish bank holiday. The contract size is 100 times the underlying index, with a tick size of 0.25 SEK.

The OMXS 30 index futures market offers contracts with varying maturities. Traders can access at least three futures contract series at any time, with different expiration dates ranging from one to three months. Contract series expire on the third Friday of the expiration month if it is a Swedish bank day, while if it is not, or if it is a half trading day, they expire on the preceding bank day. A new expiration month series is introduced four bank days before the expiration of the previous futures series. At maturity, the futures contracts are settled in cash.

Apart from regular futures contracts, traders can engage in calendar spreads trading for the OMXS 30 index futures. The OMXS 30 Roll facility is a standardized combination of trades in the futures that implements a calendar spread strategy by selling the nearby contract and simultaneously buying the second nearby contract, or vice versa. The exchange creates the combinations automatically, with a smaller tick size of 0.05 SEK compared to 0.25 SEK for individual regular futures contracts. Calendar spread trading is typically only active during expiration weeks.

3.2 Data

To empirically test the model predictions, we gather detailed trade and quote records for the OMXS 30 index futures from the LSEG Tick History (TH) database. These records are time-stamped to the nearest microsecond. The quote data provide updates on the best bid and ask prices, as well as the corresponding number of futures contracts available for trading (i.e., the best bid and ask sizes). Trade data have the transaction prices and volumes for each trade.

Additionally, we obtain intraday five-second data for the OMXS 30 index futures from TH, which include the best bid and ask prices, as well as the best bid and ask sizes at every five-second interval.

To determine which futures transactions are carried out by HFTs, we utilize data from the Transaction Reporting System (TRS). Under the Markets in Financial Instruments Directive (MiFID), all financial institutions regulated by national financial supervisory authorities within the European Union are required to report their transactions to TRS. We access these transaction data through Sweden's financial supervisory authority, Finansinspektionen. The TRS dataset provides details on trade prices and volumes, including buy/sell indicators, as well as identifiers for the trading firms responsible for each transaction.

We use the firm identifiers to distinguish between proprietary trading firms and brokerage firms. Following the approach of Baron, Brogaard, Hagströmer, and Kirilenko (2019), we classify firms that are members of the Futures Industry Association's European Principal Traders Association (FIA EPTA)—an industry group representing principal trading firms—as HFTs (see Hou et al., 2024, for a similar classification).

To avoid possible issues of extra volatility or trading volume around the opening and closing auctions for the OMXS 30 index futures, we exclude the data during the first five and last five minutes of the continuous trading session. Therefore, our empirical analyses employ trades and quotes arriving between 9:05 AM and 5:20 PM during continuous trading.

The sample period in our study spans from January 4, 2016, to December 29, 2017, excluding half trading days. Following Andersen et al. (2018), we focus on the nearby futures contract (the one closest to maturity), which is the most actively traded contract. Following Hou, et al. (2024), we further exclude the trading days during the expiration weeks (within five trading days to the maturity) to avoid potential issues of effects due to calendar spread trading, when futures traders roll over the nearby contract to the next contract. The final sample period consists of 413 trading days.

3.3 Variables

Our theoretical model proposes the regression according to Eq. (7). In this section, we explain in detail how we measure the variables bid—ask spread, variance, and the number of trades for the regression and to test the restrictions that the theoretical model put on the regression coefficients.

We adopt the methodology outlined in Andersen et al. (2018) and obtain all the variables for each intraday interval within a given day. Our sample begins at time 0 and covers D trading days, each day comprising T intraday intervals with equal length. Therefore, the entire sample consists of the consecutive non-overlapping intervals $\tau = 1, \dots, D \times T$. To identify a specific trading day and intraday interval, we use the double-index notation (d, t) where $d \in D = \{1, \dots, D\}$ represents the trading day, and $t \in T = \{1, \dots, T\}$ denotes the intraday interval.

Following Hou et al. (2024), we choose the intraday interval to be five minutes since only 0.26% of the intervals result in a zero measured variance (corresponding to an unchanged mid-point price during an interval), and thus, produce missing values of log variance.[10]

Quoted tick size adjusted bid—ask spread

[10] Selecting, e.g., one-minute intervals instead produces 9.76% missing values of log variance.

As in Li and Ye (2023), we measure RSP_τ from Eq. (6) as the average five-second quoted bid—ask spread netting the minimum tick size of SEK 0.25 for each τ, or alternatively, for each five-minute interval t, on day d, scaled by the mid-point futures price, according to:

$$RSP_\tau = RSP_{d,t} = \sum_{i=1}^{60} \frac{\left[\left(ask_{i,d,t} - bid_{i,d,t}\right) - 0.25\right]}{60 \times mid_{i,d,t}} \tag{9}$$

where $bid_{i,d,t}$ and $ask_{i,d,t}$ are the best bid and ask prices, and $mid_{i,d,t}$ is the corresponding midpoint of the quoted futures prices, prevailing at the end of the five-second interval i, within the five-minute interval t, on day d.

Return variance

We measure the return variance from five-second squared midpoint returns for each τ, or alternatively, for each five-minute interval t, on each day d, according to:

$$Variance_\tau = Variance_{d,t} = \sum_{i=1}^{60} \left[\log(mid_{i,d,t}) - \log(mid_{i-1,d,t})\right]^2. \tag{10}$$

Number of trades

Finally, we assume that trades recorded in TH with the same microsecond stamp are generated by a same single market order, and we sum the volume of these trades to represent the volume of this single market order following Xu (2014). In addition, we exclude trades which are executed within the spread, or with a transaction price that is off the tick size grid. We further exclude those trades which are identified to be calendar spread trades or block trades.[11] Finally,

[11] Trades with a volume in excess of 1,000 futures contracts which cannot be matched with the limit order book information before and after the trade are considered to be block trades executed outside of the limit order book.

we count the total number of trades, $Trades_\tau$ (or $Trades_{d,t}$), taking place within each τ, or within the five-minute interval t, on day d.

3.4 Descriptive statistics

Table 1 presents descriptive statistics for average liquidity, return volatility, trading volume, number of trades, trade size, and futures price on both a daily (Panel A) and an intraday basis (Panel B). On average, the absolute quoted futures bid-ask spread is 0.269 SEK, slightly above the minimum quoted spread level 0.25 SEK implied by the minimum tick size restriction. The relative quoted bid-ask spread equals 1.82 basis points on average. If we compare the min, max and standard deviation of quoted spreads in the daily setting to the corresponding statistics in the intraday setting, we observe that spreads are more volatile across days than across intraday periods although they are still very tight on average. Similarly, volatility exhibits more variations across days than across intraday periods and the mean futures return volatility is 0.12 on an annual basis. For the trading volume and the number of trades, we see the opposite, that the standard deviation is larger on intraday basis than that on daily basis. The average trading volume amounts to 108 MSEK per five-minute period, with 37.9% involving HFTs on at least one side of the trades. Furthermore, within a span of five minutes, typically about 117 trades are executed, with an average trade size of six contracts per trade. The mean futures price fluctuates in the range 1,256 and 1,677 across 413 trading days in our sample with a mean of 1,493.

Insert Table 1 here

Figure 1 presents the average quoted bid—ask spread in SEK and basis points across intraday five-minute intervals. The average spread in SEK is 0.269 SEK, close to the minimum tick size 0.25 SEK. The spread starts at a high level in the beginning of the day and gradually goes down over the trading day. The spread spikes at 2:30 PM and at 4:00 PM coincide with U.S. macroeconomic announcements at 8:30 ET and 10:00 ET and their associated uncertainty. Then the spread finally declines to a low level before closing. The relative spread moves in a similar pattern around an average of 1.82 bps.

Insert Figure 1 here

Figure 2 illustrates the average realized variance and number of trades across the intraday five-minute intervals. The variance and number of trades are highly correlated. Both variables follow a U-shaped curve over the trading day: the volatility and number of trades are at the peak after the opening, decline to the lowest level in the middle of day, then bounce back when the closing hour is approaching. Figure B.1 in Appendix B further illustrates the high linear correlation between volatility and number of trades.

Insert Figure 2 here

Figure 3 shows the average quoted bid—ask spread in SEK and basis point across trading days. Again, the average quoted spread in SEK is slightly above the minimum tick boundary 0.25. The relative spread declines from around 2.0 bps in January 2016 to about 1.6 bps in December 2017 as the index futures price increases over time. In Figure 4, the daily realized variance and the number of trades move closely with each other. Figure B.2 in the Appendix further illustrates the strong positive linear correlation between volatility and number of trades in logarithms.

Insert Figure 3 here

Insert Figure 4 here

In the spirit of our theoretical model, we decompose the relative bid—ask spread into two components: the tick size component and the lot size component. According to Eq. (1), the expected tick size component of the absolute spread is equal to Δ. Hence, to estimate the tick size component of the relative spread, we divide Δ with the futures price in each interval, and take the average across intervals. We estimate the lot size component as the corresponding average difference between the observed spread in each interval and Δ, divided by the futures price.

As is summarized in Table 1, the tick size component is the major driver of the spread whereas the lot size part is rather small in both daily and intraday settings. This is also seen in Figure 5 (6) where we illustrate the decomposition on an intraday (a daily) basis.[12] Moreover, the quoted spread is often binding at one tick. According to our theoretical model, a higher futures price would reduce futures transaction costs through the tick size restriction while increase transaction costs through the lot size restriction. Since, as our results show, the bid—ask spread is binding with the minimum tick size, while the trade size is not binding with the lot size, our policy recommendation for the futures exchange is to perform a "reverse index split" by increasing the index multiplier of the futures contract. An alternative way to achieve a similar effect would be for the regulator to reduce the minimum tick size in the futures market.

Insert Figure 5 here

Insert Figure 6 here

4. Regression results

[12] Note that the relative quoted spread varies more across days than across intraday time periods. This is due to the larger variability of the futures price across days than across intraday time periods (see last row of Table 1).

This section presents our main empirical results from the regressions of quoted bid—ask spread and the empirical test of our model. Table 2 (Panel A) reports the regression results according to Eq. (8) using both the intraday and the daily aggregation.[13] In the intraday regression, the estimated coefficient β_1 is -0.936, which is very close to the theoretical prediction of -1. This is confirmed with a t test if β_1 equals to -1. Table 2 holds the test results that the test statistic $t(\beta_1 = -1)$ is 0.709. Thus, we cannot reject the null hypothesis that the coefficient β_1 is equal to -1. Similarly, in the daily aggregation setting, the estimated β_1 equals -0.997, and the corresponding test statistic for the null hypothesis that $\beta_1 = -1$ is 0.031. Again, we fail to reject the null hypothesis, confirming that β_1 aligns with the theoretical prediction. In addition, we perform an F test for the joint null hypothesis that $\beta_1 = -1$ and $\beta_0 = 0$ and obtain the F statistics of 0.241 in the intraday setting and 0.002 in the daily setting. These results further support the consistency of the estimated coefficients β_0 and β_1 with theoretical predictions.

Insert Table 2 here

To investigate if our main results depend on HFT activity, we separate our sample period into days with high and low HFT activity (higher and lower the average HFT volume, respectively). In Table 2, Panel B presents regression results for days with high HFT activity and Panel B shows the corresponding results for days with low HFT activity. In each panel, we cannot reject the joint null hypothesis that $\beta_1 = -1$ and $\beta_0 = 0$. Hence, we conclude that our results are robust with respect to HFT activity.

[13] Table B.1 in Appendix B reports the Augmented Dickey-Fuller (ADF) test results for stationarity of the regression variables. In the intraday setting, we cannot reject the non-stationary hypothesis for each variable in levels, but, after taking the first difference of each variable, we can reject each hypothesis. In the daily aggregation setting, we can reject each hypothesis of non-stationarity in both levels and first differences.

As another robustness check, we perform regression analyses for both short and long futures contract maturity. In Table 2, Panel D contains regression results for when the futures contract has $\leqslant 18$ days to maturity, while Panel E has corresponding results for when maturity is $\geqslant 21$ days long. In each regression, we cannot reject the joint null hypothesis that $\beta_1 = -1$ and $\beta_0 = 0$. Therefore, we argue that our results are valid for both short and long futures contract maturity.

The regression results provide clear evidence supporting our theoretical model. We cannot reject the hypothesis that the restrictions imposed by the model hold in both the daily and intraday aggregation settings. In fact, the estimated regression coefficients β_0 and β_1 are remarkably close to the theoretical values 0 and -1, respectively. We interpret the empirical support as evidence that the model works well, and that it could be used to analyze how restrictions on minimum tick size and minimum lot size affect futures transactions costs and market liquidity. The empirical support also validates the usage of the theoretical model as a tool for policy recommendations.

5. Concluding remarks

In this paper, we study the role of restrictions on the minimum tick size and the minimum lot size for determining the bid—ask spread for futures. We develop a theoretical model for the purpose, test the model empirically, using data on OMXS 30 index futures that are traded on the Nasdaq Stockholm exchange, and provide policy suggestions for the exchange on how to reduce futures market transactions costs.

Our theoretical model produces several results. First, given the tick size and the lot size, the bid—ask spread can be decomposed into a component driven by the tick size restriction and another component driven by the lot size restriction. Since the tick size driven component is equal to the minimum tick size, it is straightforward to perform the decomposition of the bid—ask spread empirically. The decomposition highlights which restriction is more important than the other for transactions costs and points the exchange towards the correct channel through which to lower transactions costs: the tick size channel or the lot size channel.

The second result from our model is that the futures exchange can minimize futures transactions costs by choosing the optimal futures price that balances the tick size restriction with the lot size restriction. Thus, the futures price is the primary tool for reducing transactions costs when the futures market is exposed to both tick size and lot size restrictions. But it is tricky to use since an increase in the futures price would increase transactions costs through the lot size channel and decrease transactions costs through the tick size channel, and vice versa for a futures price decrease. From a policy recommendation perspective, once the exchange knows whether to choose the tick size channel or the lot size channel for reducing transactions costs, it can decide whether to decrease or increase the futures price.

Third, our model produces an expression for the futures bid—ask spread, given the lot size and tick size restrictions, as a function of futures volatility and trading activity. The expression enables us to evaluate the empirical accuracy of the model. Empirical accuracy is important since it enhances the usefulness of the model as a tool for policy makers.

We use data from the OMXS 30 index futures that are traded on Nasdaq Stockholm and regression analysis to estimate the relationship between the futures bid—ask spread, futures volatility, and trading activity, implied by the model. The empirical results strongly support the model. Moreover, the empirical analyses show that the tick size restriction is binding whereas the lot size restriction is not. Hence, for a futures exchange where these conditions are met (for example, the Nasdaq Stockholm futures exchange), our recommendation is to choose the tick size channel towards lower transactions costs and, thus, consider an increase in the futures price. This can be accomplished by an increase in the index multiplier of the futures contract. Of course, a direct lowering of the minimum tick size could also do the job.

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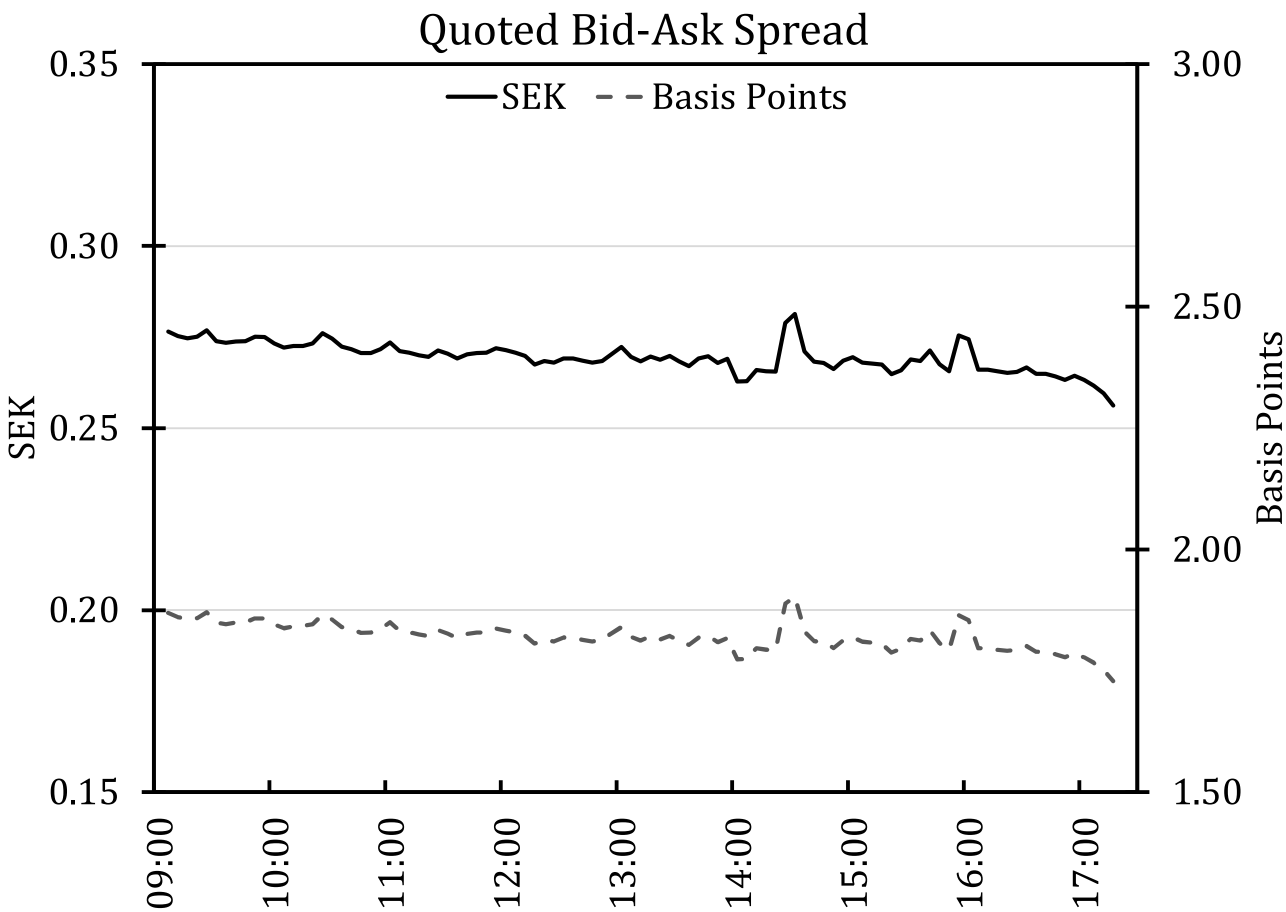

Figure 1: Intraday quoted bid-ask spread.

The figure shows the actual average quoted futures bid-ask spread in SEK and basis points, relative the concurrent midpoint quote, for each five-minute interval across all trading days in the sample.

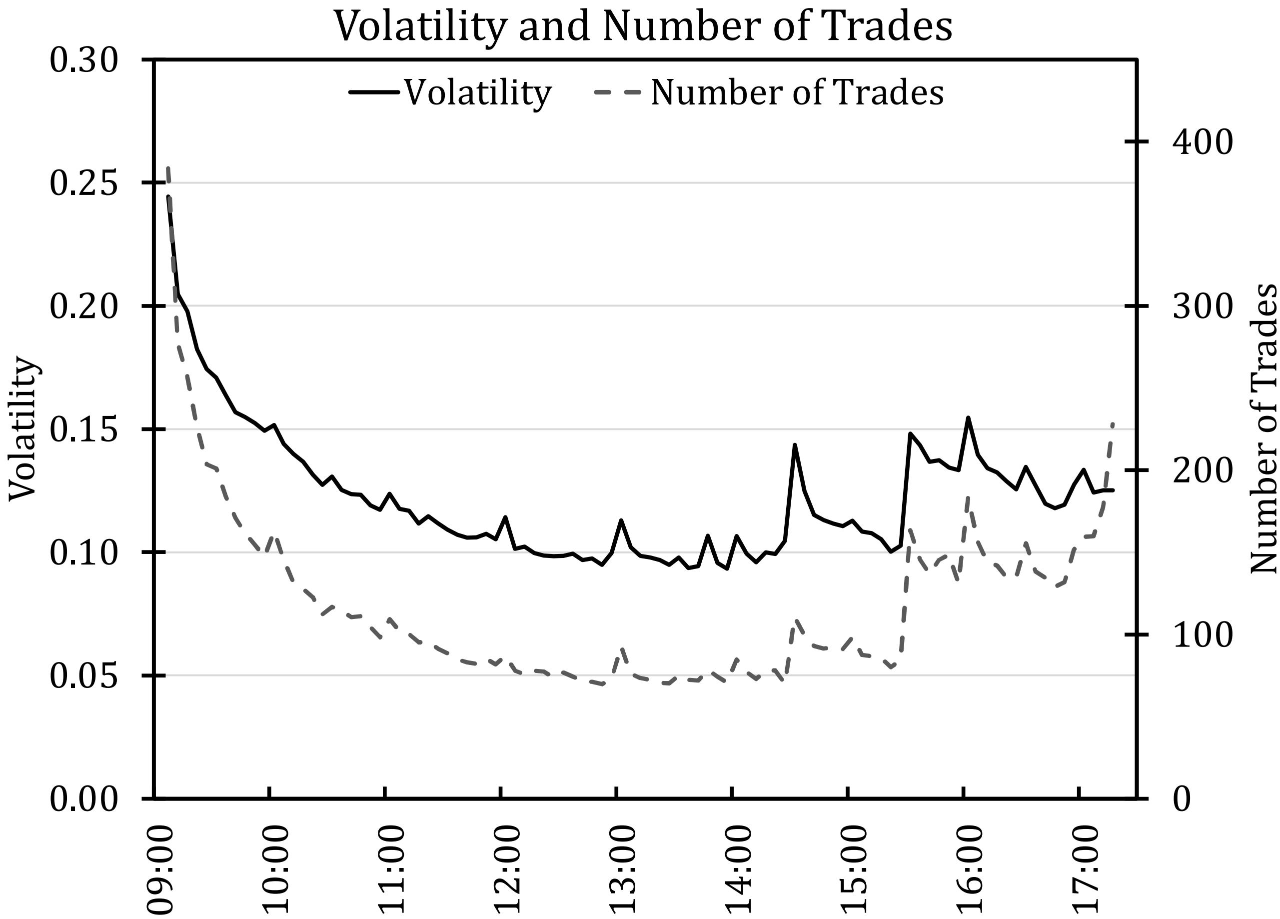

Figure 2: Intraday volatility and number of trades.

The figure shows the average futures return volatility, and the average number of futures trades, for each five-minute interval across all trading days in the sample.

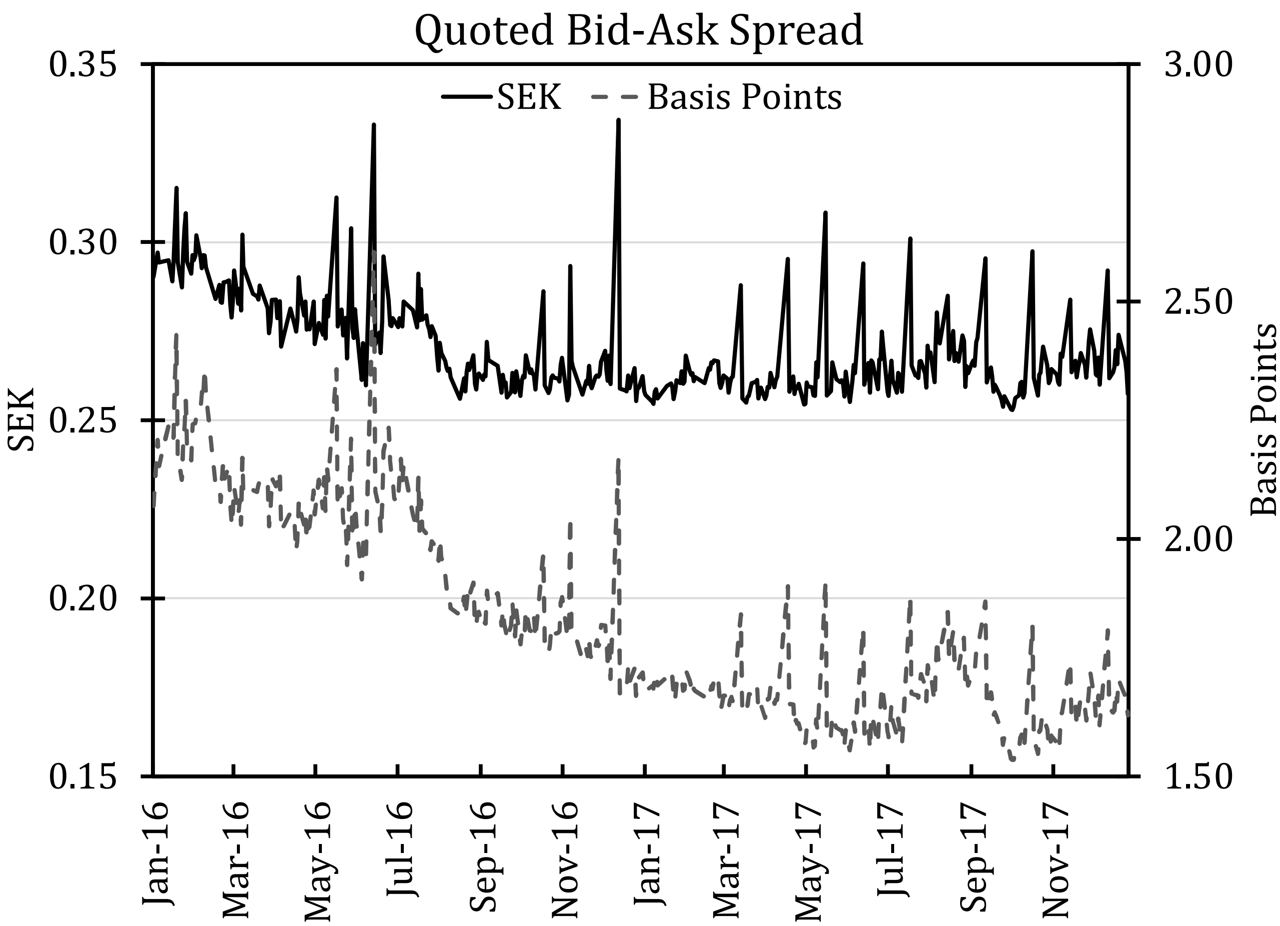

Figure 3: Daily quoted bid-ask spread.

The figure shows the actual average quoted futures bid-ask spread in SEK and basis points, relative the concurrent midpoint quote, for each trading day across all five-minute intervals in the sample.

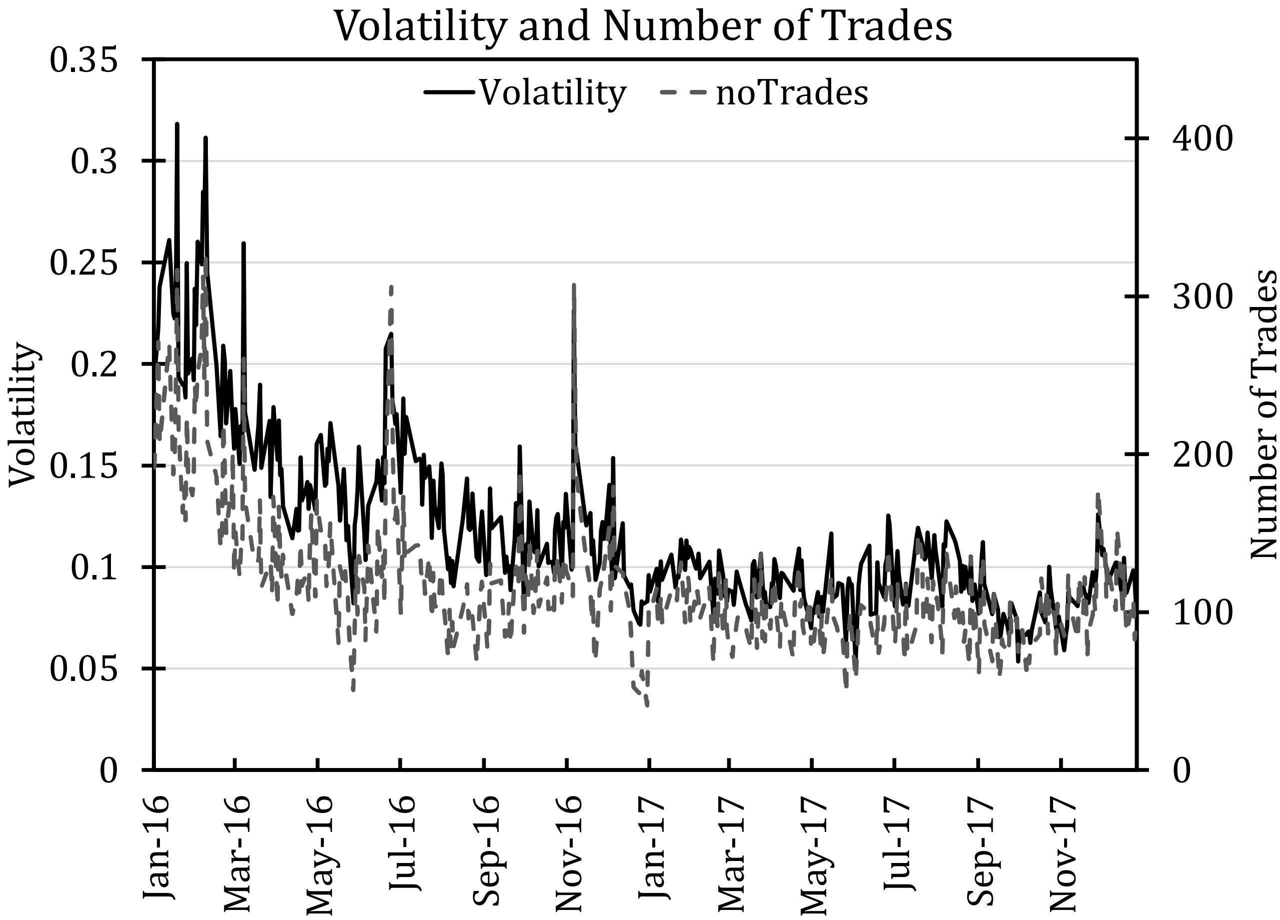

Figure 4: Daily volatility and number of trades.

The figure shows the average futures return volatility, and the average number of futures trades, for each trading day across all five-minute intervals in the sample.

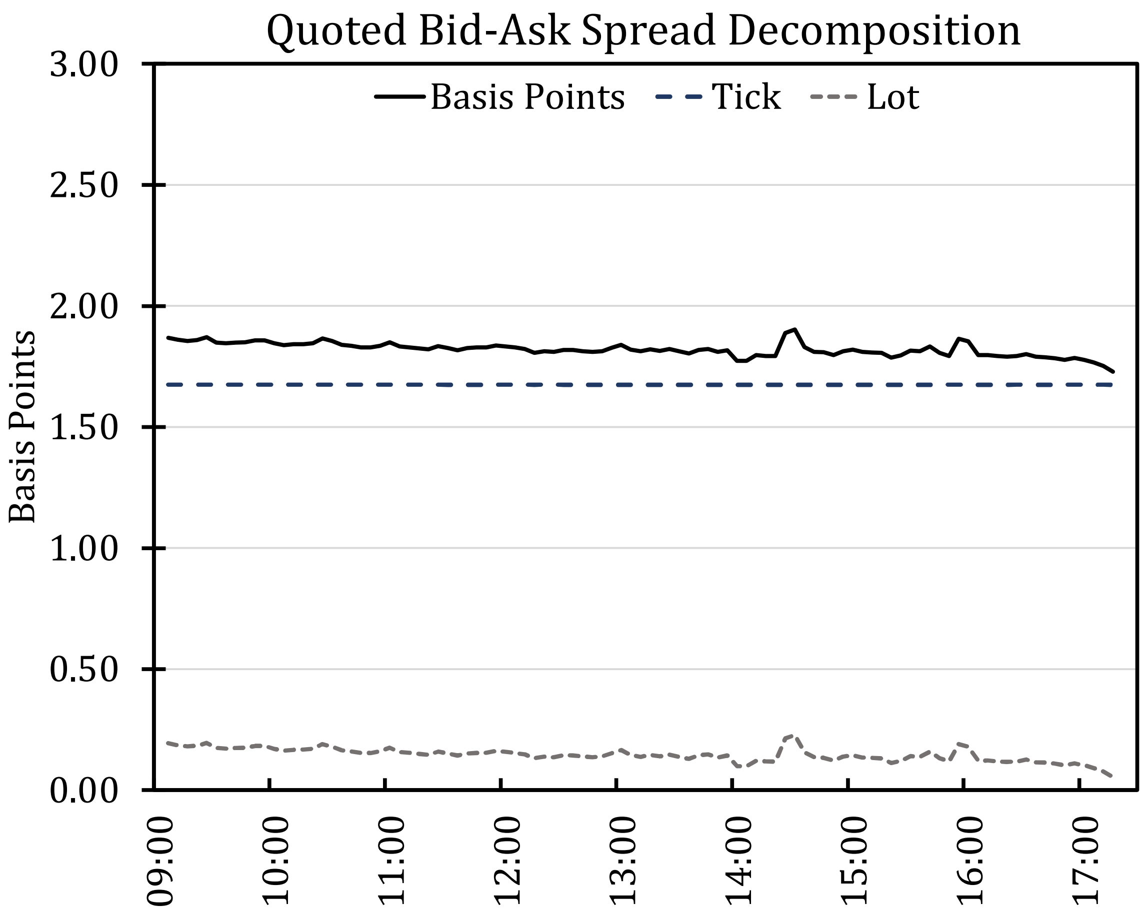

Figure 5: Intraday quoted bid-ask spread decomposition.

The figure shows the decomposition of the intraday average relative quoted futures bid-ask spread from Figure 1 into the part that is due to the tick size restriction (Tick) and the part that is due to the lot size restriction (Lot).

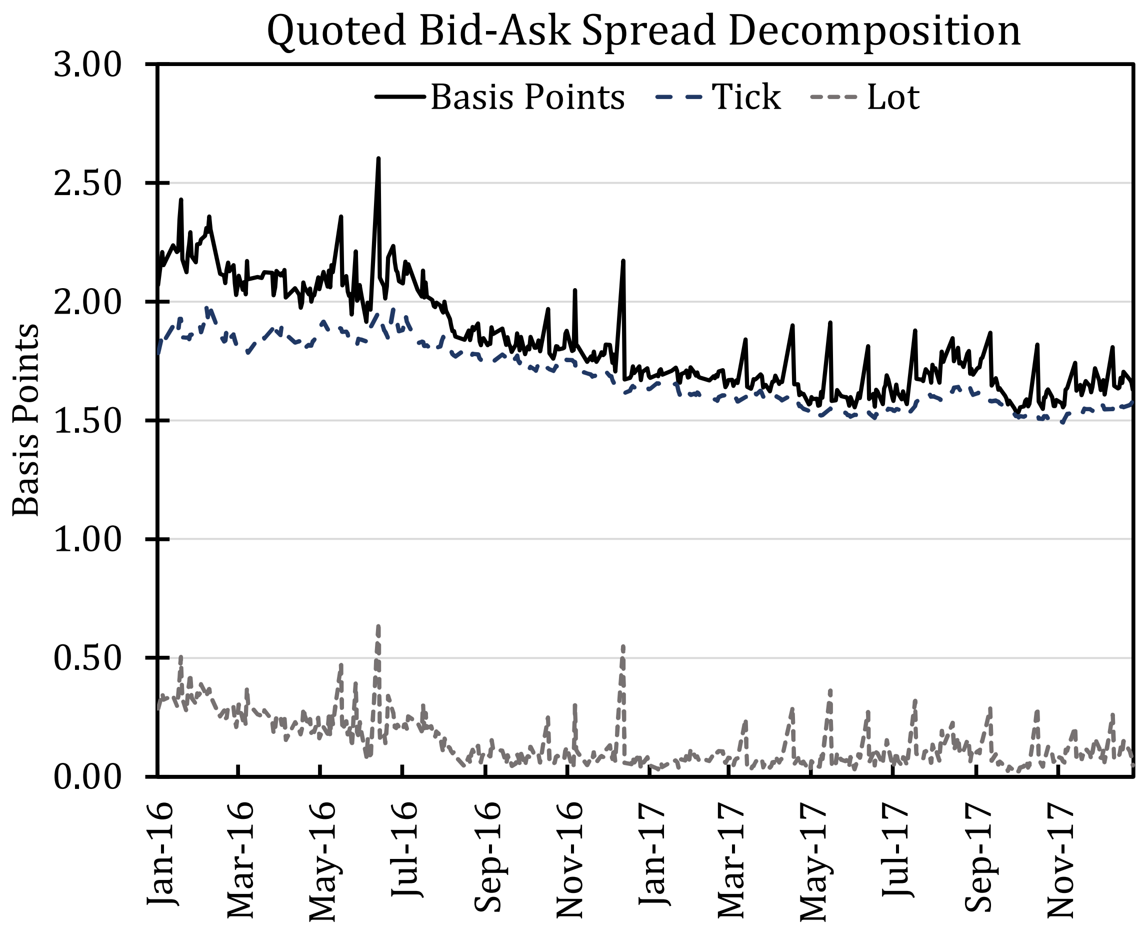

Figure 6: Daily quoted bid-ask spread decomposition.

The figure shows the decomposition of the daily average relative quoted futures bid-ask spread from Figure 1 into the part that is due to the tick size restriction (Tick) and the part that is due to the lot size restriction (Lot).

Table 1: Descriptive statistics: Daily and intraday aggregation

	Panel A: Daily				Panel B: Intraday			
	Mean	Min	Max	St. Dev.	Mean	Min	Max	St. Dev.
Absolute quoted spread	0.269	0.253	0.334	0.013	0.269	0.256	0.281	0.004
Relative quoted spread	1.820	1.535	2.604	0.212	1.820	1.729	1.903	0.028
Tick size component	1.685	1.491	1.990	0.133	1.675	1.674	1.675	0.000
Lot size component	0.135	0.017	0.649	0.099	0.146	0.054	0.228	0.028
Volatility	0.117	0.053	0.318	0.043	0.122	0.093	0.244	0.026
Volume	107.998	27.763	304.805	36.814	108.002	58.964	361.112	51.753
HFTs share of Volume	0.379	0.018	0.539	0.066	0.379	0.287	0.416	0.022
No. of Trades	117.334	41.091	328.061	43.626	117.340	69.683	383.699	51.972
Trade size	6.103	3.592	8.446	0.727	6.103	5.536	7.467	0.416
Futures price	1,492.815	1,256.157	1,676.661	115.416	1,492.812	1,492.100	1,493.068	0.170

The table presents descriptive statistics for the nearby futures contract, which is defined as the most actively traded regular futures contract on a specific day. The data include 413 days and 99 five-minute intervals per day, from 9:05 to 17:20. Half trading days are excluded. Contract maturity spans from 36 days to 7 days before the expiration day. The descriptive statistics are presented across 413 trading days for each five-minute interval (Panel A), and across 99 five-minute intervals for each trading day (Panel B). Absolute quoted spread is the average quoted bid-ask spread, i.e., best ask price minus best bid price, observed every five seconds, per five-minute interval. Relative quoted spread is the absolute quoted spread divided by the midpoint between the best ask price and the best bid price, observed every five seconds, per five-minute interval, reported in basis points. Tick size (Lot size) component is the part of Relative quoted spread associated with the tick size (lot size) restriction. Volatility is the realized volatility, calculated from the sum of five-second squared midpoint returns during each five-minute interval, averaged across all observations, and reported in annualized terms. Volume is the SEK volume of futures contracts traded per five-minute interval (in millions). HFTs share of Volume is the share of volume with a high-frequency trading firm on at least one side of the transactions. No. of Trades is the number of trades per five-minute interval. Trade Size equals the number of futures contracts per trade and per five-minute interval. Futures price is the average midpoint between the best ask price and the best bid price, observed every five seconds, per five-minute interval.

Table 2: Regression results

Aggregation	N. Obs	β_0	β_1	se(β_0)	se(β_1)	$\bar{R}^2$	$t\,(\beta_1 = -1)$	$F\,(\beta_0 = 0, \beta_1 = -1)$
Panel A: All Data								
Intraday	98	0.001	-0.936	0.004	0.091	0.598	0.709	0.241
Daily	412	-0.001	-0.997	0.007	0.109	0.371	0.031	0.002
Panel B: High HFT trading activity								
Intraday	98	-0.000	-0.937	0.011	0.093	0.563	0.676	0.234
Daily	206	-0.020	-0.804	0.020	0.119	0.265	1.644	2.312
Panel C: Low HFT trading activity								
Intraday	98	-0.001	-0.913	0.011	0.098	0.494	0.886	0.393
Daily	206	0.014	-1.162	0.029	0.116	0.422	-1.400	1.069
Panel D: Short maturity ($\leq$ 18 *days*)								
Intraday	98	0.000	-0.977	0.011	0.084	0.543	0.280	0.040
Daily	220	0.001	-0.890	0.020	0.119	0.474	1.084	0.588
Panel E: Long maturity ($\geq$ 21 *days*)								
Intraday	98	-0.001	-0.870	0.001	0.104	0.523	1.254	0.787
Daily	192	-0.005	-1.156	0.031	0.147	0.317	-1.064	0.567

The table presents results from regressions according to Eq. (8): $d(log(RSP_\tau) - log(Variance_\tau)) = \beta_1 d(log(Trades_\tau)) + \beta_0 + \varepsilon_\tau$, where RSP_τ is the average relative quoted bid-ask spread, adjusted for the minimum tick size, in interval τ, $Variance_\tau$ denotes the average realized variance in interval τ, $Trades_\tau$ is the average number of trades in interval τ, β_0 is a constant, and ε_τ is a residual with zero mean. The variables are defined in Table 1. The data include 413 trading days, and 99 intraday (five-minute) intervals, from 9:05 to 17:20. For the intraday (daily) regression, each intraday variable observation is obtained as an average across the 413 trading days (the 99 five-minute intervals per day). Regressions are performed for the nearby regular futures contract using days in the sample with 7-36 days until the expiration day. The nearby contract is defined as the most actively traded regular futures contract on a specific day. Standard errors are corrected for heteroskedasticity and autocorrelation in the residuals (up to 10 lags) according to Newey and West (1987). Panel A contains the results for all data. We also divide our sample period into two equal parts: one representing high HFT trading activity (Panel B: 206 trading days) and the other representing low HFT trading activity (Panel C: 206 trading days). For the intraday (daily) regression, each intraday variable observation is obtained as an average across the 206 trading days (the 99 five-minute intervals per day). Similarly, we divide the sample into two equal parts with respect to futures contract maturity: one with short maturity (Panel D: 220 trading days with $\leq$ 18 days to maturity) and the other with long maturity (Panel E: 220 trading days with $\geq$ 21 days to maturity).

Appendix A

Uninformed traders arrive randomly at a Poisson process with intensity λ_I. Suppose that an uninformed trader wants to trade a parent order of futures of x index units with the arrival intensity $\lambda_I = xv/pqL$, where q is the choice variable. The market maker observes q and sets bid-ask quotes B_t^q and A_t^q for futures lots of size qL that are multiples of Δ.

Assume that $\Delta = 0$ as in Budish et al. (2015). In this case, the market maker quotes a competitive relative bid-ask spread $s_t^{tot} = S_t^{tot}/p_t = S_t^L/p_t = s_t^L$ that is entirely due to the lot size restriction. An informed trader will trade if the jump size σ is larger than $s_t^L/2$ and we denote the likelihood for this to happen $Pr(\sigma > s_t^L/2)$. Thus, the arrival intensity for informed trading is $\lambda_S = \lambda_J Pr(\sigma > s_t^L/2)$.

The expected dollar revenue for the market maker when she trades with an uninformed trader is:

$$\lambda_I qLp_t \frac{s_t^L}{2}. \tag{11}$$

The corresponding expected dollar loss from trading with an informed trader is:

$$\lambda_S qLp_t \left(\sigma - \frac{s_t^L}{2}\right). \tag{12}$$

Equating the revenue and the loss yields the following expression for the bid-ask spread:

$$s_t^L = \frac{2\sigma\lambda_S}{\lambda_I + \lambda_S}. \tag{13}$$

The expression in Eq. (13) is the same as in Budish et al. (2015). Li and Ye (2023) instead obtain a corresponding expression equal to $s_t^L = 2\sigma\lambda_S L/(\lambda_I h + \lambda_S L)$, where h is the number of outstanding shares in their model of equity trading. Since $h >> L$, this expression is very small, and, consequently, the market makers in the model by Li and Ye (2023) are hardly exposed to informed trading.

Assume instead that $\Delta > 0$ and that prices and quotes occur in increments of Δ $(\Delta, 2\Delta, 3\Delta, \ldots)$. This is the main situation in Li and Ye (2023) in which the market maker cannot quote the competitive bid-ask spread S_t^L unless it conforms to the tick size grid. Instead, the market maker must round up the competitive ask quote to the closest tick above it and round down the competitive bid quote to the closest tick below. Accordingly, which is shown in Lemma 1 in Li and Ye (2023), the bid—ask quotes B_t^q and A_t^q are:

$$B_t^q = p_t - S_t^L/2 - [\Delta - mod(p_t - S_t^L/2, \Delta)], \tag{14}$$

and

$$A_t^q = p_t + S_t^L/2 + [\Delta - mod(p_t + S_t^L/2, \Delta)], \tag{15}$$

where $mod(p_t + S_t^L/2, \Delta)$ denotes the remainder of dividing $p_t + S_t^L/2$ by Δ.

To avoid losses, the market maker cannot quote a tighter spread with more aggressive quotes than the ones in Eq. (14) and (15). Hence, the bid—ask spread $S_t^{tot} > S_t^L$, with the difference being $S_t^\Delta > 0$. Li and Ye (2023) proves in their Proposition 2 that the widening effect, i.e., S_t^Δ, is equal to Δ in expectation. To sketch the proof, the authors first decompose the competitive spread as $S_t^L = a\Delta + b$, where $a = 0, 1, 2, \ldots$ denotes how many ticks the spread comprises, and

$b = mod(S_t^L, \Delta)$ is the residual that is smaller than one tick. Depending on the relative position of p_t within the tick grids, which is denoted $mod(p_t, \Delta)$, the Proposition 2 of Li and Ye (2023) shows that S_t^Δ is either equal to $\Delta - b$ or $2\Delta - b$. Moreover, if $p_t \gg \Delta$, $mod(p_t, \Delta)$ is uniformly distributed. Then, the likelihood of S_t^Δ being equal to $\Delta - b$ is $(\Delta - b)/\Delta$, and the corresponding likelihood of S_t^Δ being equal to $2\Delta - b$ is b/Δ, which makes the expected value of S_t^Δ equal to Δ. Hence, the decomposition of the expected bid—ask spread becomes the expression in Eq. (1).

Appendix B

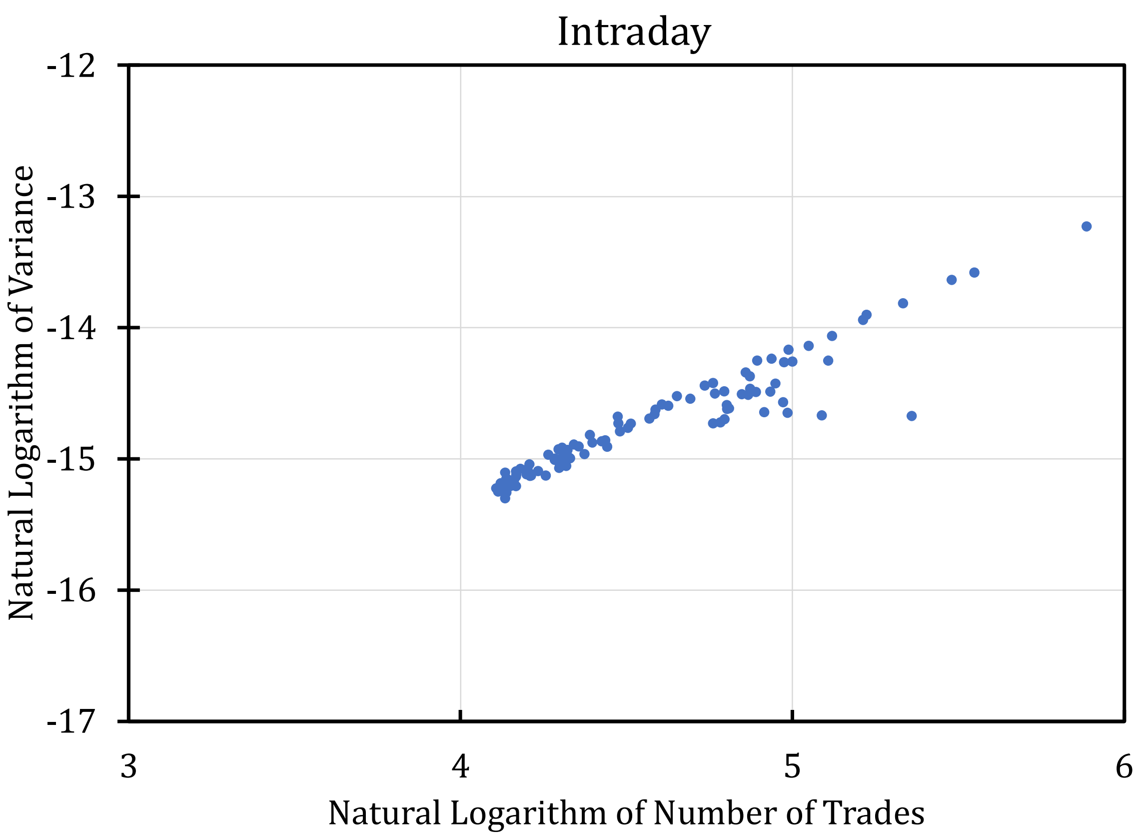

Figure B.1: Intraday variance and the number of trades.

The figure shows the intraday scatter plot of variance and the number of trades (in logarithms).

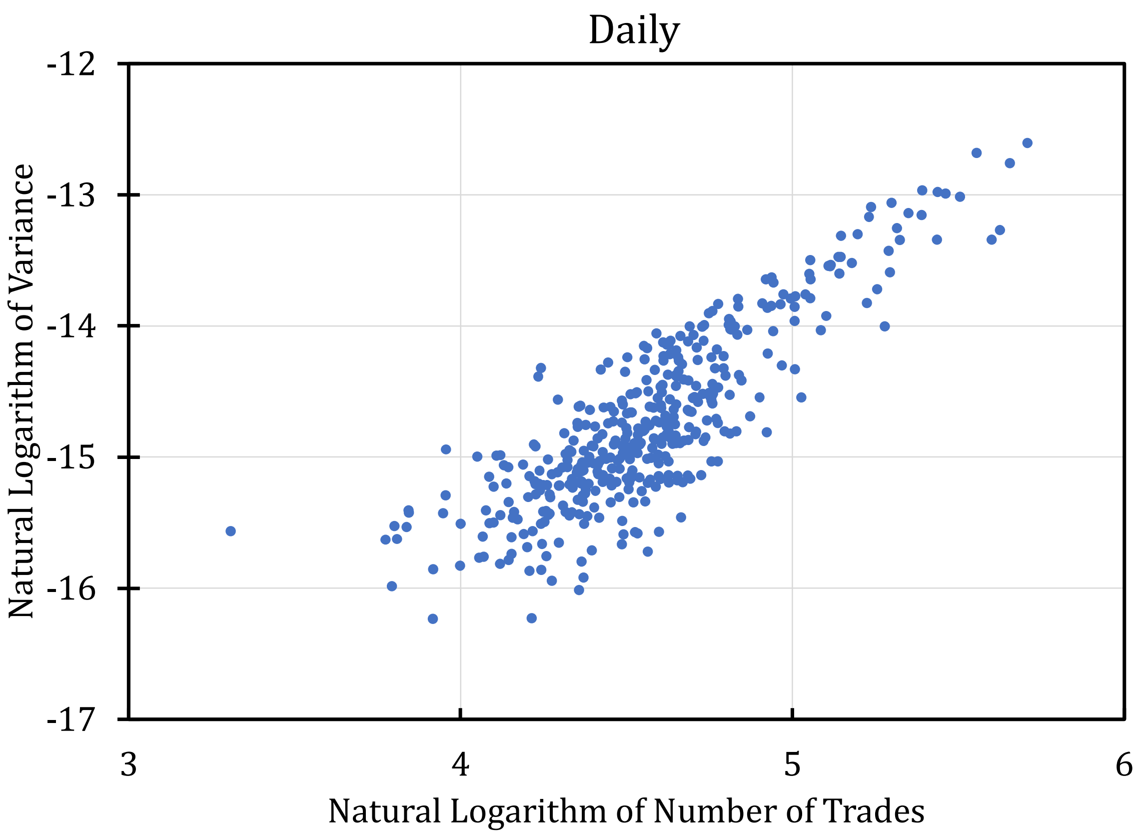

Figure B.2: Daily variance and the number of trades.

The figure shows the daily scatter plot of variance and the number of trades (in logarithms).

Table B.1: Augmented Dickey-Fuller (ADF) test results

	Intraday			Daily		
	N. Obs	ADF	p-value	N. Obs	ADF	p-value
$log(RSP_\tau) - log(Variance_\tau)$	99	-1.353	0.554	413	-4.118	<0.010
$d(log(RSP_\tau) - log(Variance_\tau))$	98	-5.719	<0.010	412	-11.435	<0.010
$log(Trades_\tau)$	99	-1.104	0.646	413	-3.954	<0.010
$d(log(Trades_\tau))$	98	-4.431	<0.010	412	-10.090	<0.010

Notes: The table presents the Augmented Dickey-Fuller (ADF) test results for stationarity of the regression variables.