Dynamic Strike Selection in Nasdaq-100 Short Put Spreads

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Short Put Spreads (SPS) allow investors to limit their downside risk in exchange for limiting upside returns. The strike prices of the long and short put options that comprise a SPS determine these upside and downside return limits. This paper compares the performance of static and dynamic methods for selecting strike prices in SPS strategies on Nasdaq-100 index options (NDX). We find that dynamic strike selection using ex-ante optimization robustly meets performance objectives such as maximizing expected profit, even with simplistic assumptions on NDX returns.
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1 Background

1.1 Short Put Spreads (SPS)

SPS are an options strategy with two primary components: (1) long put options and (2) short put options sold at a higher strike price in equal quantity and expiry. The figure below shows the payoff of NDX December 20th, 2019 SPS as of October 10th, 2019. The payoff structure of SPS allows an investor to effectively participate in returns of the underlying stock or index while maintaining a fixed upside and downside. The net payoffs of the SPS and a long position in NDX are illustrated below by the red and black lines respectively.

![Payoff of NDX December 20th, 2019 SPS as of October 10th, 2019](image)

1.2 Nasdaq-100 (NDX) Options

The Nasdaq-100® (NDX) is comprised of the 100 largest, non-financial companies listed on The Nasdaq Stock Market®. NDX options are cash-settled index options with monthly and weekly expirations as well as AM and PM settlement. NDX serves as a well-suited index on which to analyze SPS strategies due to its long history, ease of index reproducibility, and liquid, European-style options.

1.3 Studies

We simulate the hypothetical performance of several systematic NDX SPS strategies at varying strikes at the closest to 30-days to maturity using historical end of day options data from OptionMetrics, LLC. Our model SPS portfolios hold a hypothetical position in cash, earning interest at the 3-month constant maturity
treasury rate, which is assumed to be paid daily. Initial capital is $100,000,000. We assume trading occurs at 3:59PM with execution at the mid price. The study period is from March 1, 2006 to January 6, 2020. We test static strike selection performance using fixed deltas and dynamic strike selection through optimization.

2 Static Strike Selection

This static, delta-based strike selection assumes that at each option roll (one day before expiration, i.e. usually a Thursday), existing positions are closed via an offsetting position and new positions are entered with strike prices closest to target deltas. In this context, delta (Δ) is the sensitivity of a put option to changes in the NDX index. Absolute delta is often interpreted to approximate the probability of an option expiring in-the-money. For put options, the lower (negative) delta corresponds to the lower strike in the SPS. The -40Δ, 60Δ SPS strike levels are chosen to represent a benchmark SPS due to their symmetric nature being +/- 10Δ from the 50Δ center put strike. Performance metrics for various SPS delta-based strike combinations are shown below.

<table>
<thead>
<tr>
<th>Name</th>
<th>Cumulative Returns</th>
<th>Annual Returns</th>
<th>Volatility</th>
<th>Max Drawdown</th>
<th>Sharpe Ratio</th>
</tr>
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<tbody>
<tr>
<td>NDX 30-day -15/45 Delta Short Put Spread</td>
<td>103.3%</td>
<td>5.3%</td>
<td>7.7%</td>
<td>-23.6%</td>
<td>0.69</td>
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<td>2.7%</td>
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<td>0.88</td>
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<td>7.0%</td>
<td>-22.4%</td>
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<td>0.85</td>
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<td>6.6%</td>
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<td>6.5%</td>
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<td>2.4%</td>
<td>-6.1%</td>
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<td>-8.0%</td>
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<td>2.6%</td>
<td>2.3%</td>
<td>-4.3%</td>
<td>1.14</td>
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</table>

3 Dynamic Strike Selection

Optimization frameworks for the non-linear and asymmetric return streams often associated with options usually involve numerical simulations for analysis. Our optimization processes for SPS selection involves simulating NDX returns and corresponding SPS return distributions for each SPS strike combination. We then select the SPS strikes that maximize expected return and simulate entering this
new SPS at each option roll. For purposes of utilizing well-understood study assumptions and ease of reproducibility, we assume NDX follows a Geometric Brownian Motion (GBM) process, which is outlined in the appendix. Typical process assumption extensions not explored in this paper include stochastic volatility, local volatility, and other dynamic factor models for modeling the NDX price (and often implied volatility) process. Our optimization objective is to select the upper and lower SPS strike prices that maximize the ex-ante expected profit of the SPS under GBM. This process is outlined below and described in detail in the appendix.

- Generate sample returns of NDX under GBM at maturity.
- For each combination of strikes, \( K^{upper}, K^{lower} \), calculate the expected profit, \( E[\Omega] \).
- Return \( K^{upper}, K^{lower} \) that maximizes \( E[\Omega] \) subject to constraints.

The effect of GBM inputs on ex-ante optimization metrics such as expected profit is significant. For three levels of \( \mu \), (-10%, 0%, and 10%), we obtain significant variation in ex-ante metrics of expected profit. As show below in the light blue line, a ”bullish” view of 10% annual returns illustrates that the higher the upper strike (the horizontal axis), the more attractive the corresponding expected profit. This is the opposite case ”bearish” and ”neutral” views of \( \mu = -10\% \) and \( \mu = 0\% \), where expected profit is increasingly negative for higher upper strike levels.
The figures below show the ex-ante expected profit for each put-spread combination. In the first figure, the vertical axis can be interpreted as the expected profit (in $) for a given upper and lower strike combination. The GBM assumptions for this instance of an optimization, evaluated on December 16th, 2019, are $\mu = 0\%$ and $\sigma = 12.22\%$ for options with an expiry of February 21, 2020. The plot shows that the most attractive strikes have an upper strike of approximately 20 to $25\Delta$ and a lower strike of approximately $0$ to $-5\Delta$. The second figure shows the same metric (expected profit) as a function of upper and lower strike for multiple levels of $\mu$ and as a heatmap, where green corresponds to an expected profit and red to an expected loss.
3.1 Optimized Upper Strike Selection

For optimized upper strike selection, $K_{\text{lower}}$ is fixed at -40Δ and all listed values of $K_{\text{upper}}$ are evaluated in the context of the optimization. This objective aims to represent an investor who seeks to maintain a constant level of downside protection and use dynamic management to enhance upside potential.

![Profit Metrics By Upper Strike Delta](image)

*Ex-ante profit measures as a function of $K_{\text{upper}}$ in delta space. These metrics include maximum loss, the first quartile, the median, the third quartile, and the maximum gain, which are defined in the appendix.*

![Total Returns](image)

*The total returns of "Optimized NDX 30-day Short Put Spread. Upper Strike Optimized. Lower Strike = -40 Delta" shown in blue and "Benchmark (NDX 30-day -40/60 Delta Short Put Spread)" show in black.*

3.2 Optimized Lower Strike Selection

For optimized lower strike selection, $K_{\text{upper}}$ is fixed and all listed values of $K_{\text{lower}}$ are evaluated in the context of the optimization. This objective represents an
investor who is comfortable taking on more downside risk (i.e. paying more or less for long put protection) and is satisfied with a return potential represented by the 60\(\Delta\) short put strike.

The total returns of "Optimized NDX 30-day Short Put Spread. Lower Strike Optimized. Upper Strike = 60 Delta" shown in blue and "Benchmark (NDX 30-day -40/60 Delta Short Put Spread)" show in black.

The annualized returns of a static (fixed delta) and two dynamic SPS (optimized to maximize returns). Both dynamic strike selection methods were effective in enhancing annual returns, by 153 and 170 basis points respectively.

4 Conclusion

This study aims to represent a small fraction of the possibilities of optimization within options strategies. Structurally, Short Put Spreads serve as an ideal test-
ing ground for researching optimization methods and for potentially applying these methods in practice. This is primarily because while SPS have non-linear returns distributions, they also have defined risk limits that are expressible analytically. Most investors also have a strong preference to the positive equity beta inherent in SPS. This study finds that static benchmarks in options (such as -40/60 ∆) can be enhanced via simple optimization processes to maximize expected profit. Further analysis which may yield interesting findings includes evaluating different optimization objectives, testing alternative return assumption models, and optimizing position sizing of SPS.

5 Appendix

5.1 Simulated Geometric Brownian Motion

Let $S_t$ follow Geometric Brownian Motion given by the following stochastic differential equation:

$$dS_t = \mu S_t dt + \sigma S_t W_t$$

Where $\mu$ is a constant drift term and $\sigma$ is a volatility term (both annualized and expressed in %). $W_t$ is a Brownian motion, let $W_t$ be a Monte Carlo simulation drawing calculated from an inverse standard normal distribution with mean 0 and a variance of $t$.

Because $S_t$ behaves according to Geometric Brownian Motion, $S_t$ is distributed log-normally, and price returns are distributed normally (where $\phi(m, v)$ denotes a normal distribution with mean $m$ and variance $v$).

\[\ln(S_t) \rightarrow \phi[\ln(S_0) + (\mu - \frac{\sigma^2}{2})t, \sigma^2 t]\]

The standard deviation of $\ln(S_t)$ is therefor equal to $\sigma \sqrt{T}$.

The first and second moments of $S_t$ are given below:

\[E[S_t] = S_0 e^{\mu t}\]

\[Var[S_t] = S_0^2 e^{2\mu t}(e^{\sigma^2 t} - 1)\]

The Probability Density Function (PDF) of GBM is given by:

\[f(S_t|t, \mu, \sigma, S_0) = \frac{1}{2\pi S_t \sqrt{t}} exp\left[-\frac{(\ln(S_t/S_0) - (\mu - \frac{1}{2}\sigma^2) t)^2}{2\sigma^2 t}\right]\]
This framework allows for the probabilistically grounded assumptions on the generation of stochastic paths (following a GBM) which represent an agent’s subjective (”P-world”) assumption of the NDX index prices.

5.2 Optimization Process for Strike Price Selection

Below is the pseudo-coded process for selecting strike prices in a SPS.

1. For time steps \(i = 1\) to \(N\), Generate a price path where \(S_{i+1,j} = S_{i,j}e^{((r - \frac{\sigma_{K,T}^2}{2})dt + \sigma_{K,T}W_i\sqrt{dt})}
\)

\(W_i \sim N(0, 1)\)

2. For paths \(j = 1\) to \(M\), Store all \(S_{i,j}\)

3. The last value of the simulated paths \(N\) represents the quotient of calendar days until expiration and 365 in time space where a unit, \(t = 1\), represents one year and \(dt = T/N\). \(V_{S_{PS}}^T\) is the payoff of the SPS (the sum of the long and short European Put payoffs) and has the following value at maturity, \(T\):

\[V_{S_{PS}}^T(S_N) = \max[K_{lower} - S_N, 0] - \max[K_{upper} - S_N, 0]\]

4. \(\Omega\) represents the profit of the SPS for one simulated final stock price. Profit is equal to the payoff of the SPS less the initial credit or debit of the position.

\[\Omega = V_{S_{PS}}^T(S_N) - V_0(K_{upper}) - V_0(K_{upper})\]

\(E[\Omega]\) represents the expected profit of the trade for a given set of upper and lower strike prices. \(\Omega|S_N = K_{upper}\) and \(\Omega|S_N = K_{lower}\) correspond to the maximum potential SPS gain and loss respectively.

5. Solve for \(K_{upper}\) and \(K_{upper}\) that maximizes the chosen objective function, \(\max[E[\Omega]]\). In our optimization, strikes are chosen based on historical options data available that is closest to a given delta level.

6 References


7 Disclosure

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